

Collisional energy losses of heavy quarks in relativistic nuclear collisions at energies available at the BNL Relativistic Heavy Ion Collider

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We investigate the energy losses of b and c quarks at hard collisions with gluons and light quarks in quark-gluon plasma produced in central $Au + Au$ collisions at RHIC energy ($\sqrt{s_{NN}} = 200$ GeV). As the coupling constant at hard collision in the perturbative QCD is limited by Landau pole at momenta $Q^2 = \Lambda^2$, we use the analytic models of QCD, where the Landau pole is absent. We calculate the energy losses of b and c quarks in analytic model of QCD at $Q^2 \geq \Lambda^2$. At calculations we use the effective quasiparticle model. We use at calculations of total nuclear modification factor R_{AA}^{b+c} the fragmentation of b and c quarks into heavy hadrons with accounting their energy losses. We show that this factor gives the considerably suppression of heavy B and D mesons in the region of middle meanings of p_{\perp} . The experimental data [2] show, that nonphotonic electrons have the analogous suppression in the same region of p_{\perp} .

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I. INTRODUCTION

Heavy quarks, unlikely light quarks do not come to an equilibrium with the surrounding matter and may therefore play an important role in the search for properties of this matter. Produced in hard collisions, their initial momentum can be inferred from pp collisions. The deviation of the measured heavy mesons p_{\perp} distribution in AA collisions from that measured in pp collisions is usually quantified by the nuclear modification factor

$$R_{AA} = \frac{dN_{Au+Au}}{T_{AA} d\sigma_{p+p}}, \quad (1)$$

where dN_{Au+Au} is the differential electrons yield from heavy meson decays in $Au + Au$ collisions and $d\sigma_{p+p}$ is corresponding cross section in $p + p$ collisions in any given p_{\perp} bin.

In the RHIC experiments the heavy mesons have not yet been directly measured. Both the PHENIX [1] and STAR [2] collaboration have observed only single nonphotonic electrons, which have been created in the semileptonic decay of heavy mesons. Thus experimentally it is not easy separate between charm and bottom hadrons. The QCD calculations in the fixed order + next to leading logarithm (FONLL) predict [3] the theoretical uncertainty for the charm and bottom quarks p_{\perp} distribution in pp -collisions at $\sqrt{s_{NN}} = 200$ GeV. Apparently, that above $p_{\perp} = 4$ GeV/c the electrons from bottom mesons dominate the spectrum. However, uncertainty here is considerable. The nonphotonic electron yield at RHIC measurement exhibits unexpectedly large suppression in central $Au + Au$ collisions at high p_{\perp} . The observed values $R_{AA} \sim 0.2$ are smaller than originally expected. The theoretical approaches based on perturbative QCD (pQCD) calculation give much larger values

and it has been doubt, whether pQCD is the right description of this interaction. One should note that for light quarks the radiative energy loss is the most important. The heavy quarks have a smaller amount of radiative energy loss due to their large mass [4], [5].

The asymptotic freedom in QCD enable one to apply the standard perturbative formulas at large momenta transferred. However a number of phenomena are beyond such calculation. Moreover, the perturbative theory contain unphysical singularities in the expression for the running coupling, for example the ghost poles and also the unphysical cuts. Therefore by number of authors were considered the analytic models of invariant charge in QCD. The condition of analyticity follows from the general principles of local Quantum Field Theory. The analytic models contain no unphysical singularities and running coupling may be determined in the spacelike (Euclidean) region and in the timelike region by analytic continuation from spacelike region. In this work we investigate the collisional energy losses of b and c quark in quark-gluon plasma (QGP) produced in central $Au + Au$ collisions at RHIC energy. We use here the formulas for analytic invariant charge in QCD in the spacelike region. We show that the nuclear modification factor R_{AA}^{b+c} observed in the experiments is determined by hard collisions with QCD analytic charge $\alpha_{an}(Q^2)$ which have the Landau pole at $Q^2 \cong \Lambda^2$ in the perturbative QCD, but have no of this pole in analytic QCD. The vicinity $Q^2 \cong \Lambda^2$ one should consider as boundary of hard collisions in QCD.

In Sec. II we give the brief description of analytic models results, which we use in this paper, and corresponding reference.

In Sec. III we give the brief description of effective quasiparticle model and physical characteristic of expanding plasma. We calculate here energy losses $\Delta E/E$ of b and c quarks at collisions with gluons and light quarks in plasma, using the analytic coupling in QCD. We calculate also the summary losses of b and c quarks for various energies while they transverse the expanding

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plasma.

In Sec. IV we calculate the common nuclear modification factor R_{AA}^{b+c} for b and c quarks, which fragmented into B and D mesons. We use the fragmentation function b and c quarks with accounting of their energy losses. We take into account the relative B and D contribution into electrons from their semileptonic decays. We compare this modification factor with experimental data.

In Sec. V - Conclusion.

II. ANALYTIC MODELS FOR THE QCD RUNNING COUPLING

The effective coupling constant $\alpha_s(Q^2)$ is given in the lowest approximation (one loop) by the famous asymptotic formulas

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2N_f}{3}) \ln \frac{Q^2}{\Lambda^2}}, \quad (2)$$

where Λ is dimensional constant of QCD. This coupling constant have divergence at $Q^2 = \Lambda^2$. That is related to an infrared Landau pole in the running coupling, that is nonperturbative issue. In the two-loop case we have besides pole also the unphysical cut owing to the factor of type $\ln(\ln \frac{Q^2}{\Lambda^2})$. In consequence of these difficulties the number of authors use the analytic models of invariant charge in QCD. For example, some of them; [6], [7], [8], [9]. The Källén - Lehman spectral representation satisfied to condition of analyticity and it applied to the "analytization" of the renormalization group (RG) equations. We describe here briefly the analytic model proposed in the works [8], [9]. Here the solution is derived as the solution of the RG equation with the β function "improved" by the analytization procedure. This correspond to demand of analyticity of right hand side of RG equations as a whole before their solution by use of the Källén - Lehmann integral representation. At the l -loop level the analytic running coupling $\alpha_{an}^{(l)}(q^2)$ is defined as the solution of equation:

$$\frac{d \ln \alpha_{an}^{(l)}(q^2)}{d \ln q^2} = - \left\{ \sum_{j=0}^{l-1} B_j [\alpha_s^{(l)}(q^2)]^{j+1} \right\}_{an}. \quad (3)$$

The value $\alpha_{an}^{(1)}(q^2)$ at the one-loop level has here the form:

$$\alpha_{an}^{(1)}(q^2) = \frac{4\pi(z-1)}{\beta_0 z \ln z}, \quad (4)$$

where $z = \frac{q^2}{\Lambda^2}$, $\beta_0 = 11 - \frac{2n_f}{3}$. This invariant charge have no of the Landau pole in the vicinity of the point $z = 1$, namely $\alpha_{an}^{(1)}(z = 1 + \epsilon) \simeq \frac{4\pi}{\beta_0}(1 - \frac{\epsilon}{2})$ and it describe also the asymptotic for large z . The infrared enhancement at small z is in agreement with the Schwinger - Dyson equations [10]. The analytic invariant charge

(AIC) at the higher loop levels was also investigated in the works [8], [9] on the basic of the Källén - Lehmann type integral representation. It was shown that AIC possesses the higher loop stability. i.e. the curves corresponding to the two and three-loop levels are almost indistinguishable. In the present work we use analytic invariant charge in the form (4) for investigation of energy losses b and c quarks at collisions with gluons and light quarks in the quark-gluon plasma produced in central $Au + Au$ collisions at RHIC energy.

III. COLLISIONAL ENERGY LOSSES OF b AND c QUARKS IN QUARK-GLUON PLASMA

By analogy with the works [11], [12] we use the effective quasiparticle model for calculation of physical characteristics of expanding plasma. We assume the hot glue production at the early stage. This is caused by the relatively large gg cross section in comparison with qq and $q\bar{q}$ ones [13]. In the work [12] were calculated the initial temperature T_0 and time τ_0 at $\sqrt{s_{NN}} = 200$ GeV and also $\tau(T)$ for evolution of the plasma phase for $T \rightarrow T_c$ from above. For these estimations we used the formulas of the quasiparticle model for gluons and quarks densities n_g, n_q, n_s at temperature T at expanding of plasma [14]. These densities depend from thermal masses $m_g(T)$ and $m_q(T)$ of gluons and quarks. These thermal masses are connected with the thermodynamic functions of medium and they are determined by effective coupling $G^2(T)$. They describe well the lattice data at $T_c \leq T \leq 4T_c$. We use here the parametrization of effective two-loop coupling $G^2(T, T_c)$ from Ref. [14]. We investigate in this work the central $Au + Au$ collisions for nuclei modeled as a uniform cylinder with sharp edge and effective radius $R_{eff} \simeq 6$ fm, which correspond to $\tau \simeq 6$ fm and $T \simeq 183$ MeV [15]. This value of T is close to $T_c = 177$ MeV [16]. The calculation of the lepton spectrum includes initial b and c quarks distribution from perturbative QCD, heavy quarks Q energy losses, their fragmentation into heavy mesons H_Q and electron decay spectrum of heavy mesons. This cross section is schematically written as [3], [5]:

$$\frac{E d^3 \sigma(e)}{dp^3} = \frac{E_i d^3 \sigma(Q)}{dp_i^3} \otimes P(E_i \rightarrow E_f) \otimes \otimes D(Q \rightarrow H_Q) \otimes f(H_Q \rightarrow e), \quad (5)$$

where the symbol \otimes denotes a generic convolution. The term $f(H_Q \rightarrow e)$ includes the branching ratio to electrons. The initial b and c p_\perp distributions in pp collisions with account of theoretical uncertainty were computed at FONLL level in Ref. [3]. In this work we use these differential cross section $\frac{d\sigma}{dy dp_\perp}$ (look at Fig. 1).

The invariant cross section for collisions of heavy quark Q with momentum \mathbf{p} and massless partons with momen-

tum \mathbf{k} in plasma can be written in the form:

$$\begin{aligned} d\sigma &= d\mathbf{p}' d\mathbf{k}' \frac{4\alpha_s^2}{4E_p k'} \frac{|M|^2}{4J} \delta(\mathbf{p} + \mathbf{k} - \mathbf{p}' - \mathbf{k}') \times \\ &\times \delta(E_p + |k| - E_{p'} - |k'|) \\ &\equiv d\mathbf{p}' d\mathbf{k}' F(\mathbf{p}, \mathbf{k}, \mathbf{p}', \mathbf{k}', m_Q), \end{aligned} \quad (6)$$

where \mathbf{p}' and \mathbf{k}' are the momenta of outgoing heavy quarks Q and light partons in plasma, $|M|^2$ is the matrix element squared. The value $J = \sqrt{(PK)^2}$, where $PK = E_p |k| - \mathbf{p}\mathbf{k} = k(E_p - p \cos \Psi)$ and Ψ the angle between vectors \mathbf{p} and \mathbf{k} . Thus $J = \frac{S - M_Q^2}{2}$, where $S = M_Q^2 + 2|k|(E_p - p \cos \Psi)$. Using the formula (6) for cross section $d\sigma$, one should determine the relative energy loss $\frac{\Delta E}{Ed\tau}$ per unit length (for Qg collisions) from integral:

$$\int \frac{d\mathbf{k} n_g(\mathbf{k}) \omega}{(2\pi)^3 p} d\sigma, \quad (7)$$

where $\omega = E_p - E_{p'}$ (in t channel) and $n_g(\mathbf{k})$ is the gluon density in the quasiparticle model. We calculate in these equations at first the k' -integral, using three-dimensional δ function, and this gives $k' = |\mathbf{k} + \mathbf{p} - \mathbf{p}'|$. For the second δ function it is convenient to choose for massless partons the z axis along the direction \mathbf{p} of the heavy quarks, and \mathbf{k} contained in the yz plane [12], [17]. We use the second δ function for calculation of the integral I over azimuthal angle ϕ of the form:

$$I = \int_0^{2\pi} \delta(E_p - E_{p'} + k - \sqrt{A + B \cos \phi}) d\phi. \quad (8)$$

The result of calculation is:

$$I = \frac{2}{\sqrt{B^2 - A^2}}. \quad (9)$$

In values A and B we change the variables from p' and $\cos \theta$ to variables t and ω with corresponding Jacobian, and $t = 2M_Q^2 - 2(E_p E_{p'} - pp' \cos \theta)$. The formula of type (9) was used in Ref. [17] for investigation of collisional energy loss of muons with mass M in the hot QED plasma. The value $B^2 - A^2$ has here the form: $B^2 - A^2 = -a^2 \omega^2 + b\omega + c$. We use in this paper the coefficients a, b, c from Ref. [17] for the masses M_Q of heavy quarks. The value $B^2 - A^2$ is positive in interval $\omega_{\min} \leq \omega \leq \omega_{\max}$, where $\omega_{\min}^{\max} = \frac{b \pm \sqrt{D}}{2a^2}$. The discriminant $D = 4a^2 c + b^2$ is [17]:

$$D = -t[st + (s - M_Q^2)^2] \left(\frac{4k \sin \Psi}{p} \right)^2. \quad (10)$$

The condition $D \geq 0$ gives $0 \leq |t| \leq \frac{(s - M_Q^2)^2}{s}$. However we use here the analytic model of QCD with lower boundary of $-t$ in the vicinity of Landau pole $t \simeq \Lambda^2$ where the QCD running coupling for hard collisions contain no unphysical singularity. We can evaluate now the

relative energy loss $\frac{\Delta E}{E}$ for b and c quarks using the equations (6) and (7) and the ω integral:

$$\int_{\omega_{\min}}^{\omega_{\max}} \frac{\omega d\omega}{\sqrt{B^2 - A^2}} = \frac{\pi |t| p [E_p(s - M_Q^2) - k(s + M_Q^2)]}{(s - M_Q^2)^3}. \quad (11)$$

We have thus the formula (for Qg collisions):

$$\begin{aligned} \frac{\Delta E}{Ed\tau} &= \int \frac{d\mathbf{k} n_g(\mathbf{k})}{(2\pi)^3} \int_{|t|_{\min}}^{(s - M^2)^2/s} d|t| \alpha_{ac}^2(t) \pi |t| \times \\ &\times \frac{(s + M^2) \left[\frac{E(s - M^2)}{s + M^2} - k \right] |M_{Qg}|^2}{(s - M^2)^3 E 4J}, \end{aligned} \quad (12)$$

where $4J = 2(s - M^2)$, and M is M_b or M_c - the mass of b or c quark. The square of the matrix element in t channel have the form [18]: $\frac{|M_{Qg}|^2}{4J} = \frac{\bar{s} - |t|}{t^2}$ where $\bar{s} = s - M^2$. In quasiparticle model the thermal distribution of gluons is $n_g(k) = \frac{1}{\exp(\sqrt{y^2 + \frac{m_g^2(T)}{T^2}}) - 1}$, where $k = yT$, and

the thermal mass $m_g(T)$ is expressed across the effective two-loop coupling $G^2(T, T_c)$ from Ref. [14]. From equation (12) we have the condition $(E - \frac{M^2}{E - p \cos \Psi}) \geq k$. It is convenient to introduce the variable $\xi_1 = E/p - \cos \Psi$, i.e. the condition is $E - \frac{M^2}{\xi_1 p} \geq k$. We have also $\bar{s} = 2yT\xi_1 p$. From this condition and also from the condition $\bar{s}^2 \geq |t|_{\min}$ we have $k \geq \frac{\sqrt{|t|_{\min}} M}{2\xi_1 p} \left(1 + \frac{\sqrt{|t|_{\min}}}{2M} \right)$ and $\xi_1 \geq \frac{M^2}{Ep} \left(1 + \frac{\sqrt{|t|_{\min}}}{2M} \right)$. The upper limit for variable $|t|$ in formula (12) is

$$\frac{(\bar{s})^2}{s} = \frac{(2y\xi_1)^2 T^2 \frac{p^2}{M^2}}{1 + 2y\xi_1 \frac{T p}{M^2}}. \quad (13)$$

We use for calculations the mass of heavy quarks $M_b = 4.75$ GeV, $M_c = 1.3$ GeV. For example, at $p = 8$ GeV we have $E_b = 9.304$ GeV and $E_c = 8.1$ GeV. We have also $\xi_{1 \max} = (E/p + 1)$. We write down now the formula (12) for heavy quarks with the mass M with accounting of analytic invariant charge in the form (4):

$$\begin{aligned} \frac{\Delta E}{Ed\tau} &= \frac{16T}{4\pi} \frac{T^2}{pE} \left(\frac{4\pi}{\beta_0} \right)^2 \times \\ &\times \int_{\xi_{1 \min}}^{E/p+1} \frac{d\xi_1}{2\xi_1} \int_{y_{\min}}^{\frac{E}{T} \left(1 - \frac{M^2}{pE\xi_1} \right)} dy y \frac{\left[\frac{E}{T} \left(1 - \frac{M^2}{pE\xi_1} \right) - y \right]}{e^{\sqrt{y^2 - \frac{m_g^2(T)}{T^2}}} - 1} \\ &\times \frac{(2y\xi_1)^2 \frac{T^2 p^2}{\Lambda^2 M^2}}{1 + 2y\xi_1 \frac{T p}{M^2}} \int_{\frac{t_{\min}}{\Lambda^2}}^{\frac{t_{\min}}{\Lambda^2}} \frac{dx}{z} \left(\frac{z-1}{z \ln z} \right)^2, \end{aligned} \quad (14)$$

Table I: The relative energy losses for b and c quark at bg and cg collisions

$T(\text{MeV})$	$\tau(m_\pi^{-1})$	$(\frac{\Delta E}{E})_b$	$(\frac{\Delta E}{E})_c$
410.6	0.405	0.143	0.49
390	0.474	0.128	0.433
370	0.543	0.113	0.379
350	0.613	0.0993	0.329
330	0.691	0.086	0.283
310	0.825	0.0738	0.239
290	0.98	0.0618	0.199
270	1.07	0.0508	0.162
250	1.19	0.0404	0.127
240	1.26	0.0355	0.115
233	1.31	0.0324	0.1
220	1.62	0.026	0.0806
210	1.92	0.0211	0.0656
200	2.36	0.0163	0.0505
190	3.08	0.0112	0.0347
183	4.13	0.0073	0.0226

where $\xi_{1\min} = \frac{M^2}{E_p}(1 + \frac{\sqrt{t_{\min}}}{2M})$, $y_{\min} \simeq \frac{M\sqrt{t_{\min}}}{2\xi_{1p}T}$. We use here the analytic approach in QCD for investigate of energy losses of heavy quarks at hard collisions with light partons in plasma at temperature $T_0 \geq T \geq T_c$. We investigate the energy losses of b and c quarks in the region of hard collisions $|t| \geq \Lambda^2$, where the perturbative QCD is no applicable in the vicinity of Landau pole $|t| \simeq \Lambda^2$.

In the work [12] were found the physical parameters T and τ for expanding plasma at $\sqrt{s} = 200A$ GeV with initial meanings $T_0 = 410.6$ MeV and $\tau_0 = 0.405m_\pi^{-1}$. We use here the number of this parameters for calculations of $\frac{\Delta E}{E}$ for b and c quarks at different energies by formula (14). These calculations were fulfilled by means of numerically. We show in the Table I the energy losses of b and c quarks for bg and cg collisions for series of T and τ for expanding plasma (for example at $p_{b,c} = 8$ GeV). We have here at $|t|_{\min} = \Lambda^2$ for b quark ($M_b = 4.75$ GeV) $\xi_{1\min} = 0.31$, $y_{\min} = \frac{0.0064E_b}{\xi_{1p}T}$, and for c quark ($M_c = 1.3$ GeV) $\xi_{1\min} = 0.028$, and $y_{\min} = \frac{0.002E_c}{\xi_{1p}T}$, and for Landau pole $z_{\min} = 1$. The invariant analytic charge (4) in formula (14) describe also the asymptotic for large z , and we use the ordinary QCD constant $\Lambda = 200$ MeV. In the third column of the Table I we give $\frac{\Delta E}{E}$ for bg collisions, in the fourth column we give the same energy losses for cg , and then we give the summary energy losses $(\frac{\Delta E}{E})_{tot}$.

The calculation gives for summary energy losses: for b quark $(\frac{\Delta E}{E})_{tot}^b = 0.112$, and for c quark $(\frac{\Delta E}{E})_{tot}^c = 0.357$.

One should determine also the energy losses of heavy quarks at collisions with light quarks in the plasma. We assume that in the initial stage at $T = T_0$ all the quarks are nonequilibrated. With decrease of temperature (at $T \leq T_0$) the number of equilibrated quarks increase. In the work [12] we estimated the part β of equilibrated quarks at expansion of plasma. At $T \leq 233$ MeV the all

Table II: The relative energy losses for b and c quarks at bq and cq collisions

$T(\text{MeV})$	$\tau(m_\pi^{-1})$	$(\frac{\Delta E}{E})_b$	$(\frac{\Delta E}{E})_c$
410.6	0.405	0.0295	0.1013
390	0.474	0.036	0.1222
370	0.543	0.0402	0.1351
350	0.613	0.0454	0.1511
330	0.691	0.0502	0.165
310	0.825	0.055	0.1803
290	0.98	0.0612	0.198
270	1.07	0.051	0.1627
250	1.19	0.042	0.133
240	1.26	0.0381	0.12
233	1.31	0.0348	0.1085
220	1.62	0.029	0.0903
210	1.92	0.025	0.0776
200	2.36	0.0205	0.0635
190	3.08	0.0156	0.0484
183	4.13	0.0113	0.035

quarks become equilibrated. We estimate here the heavy quarks energy losses also at collisions with portion of nonequilibrated quarks. We assume that nascent quarks have the similar momentum distribution as gluons [13]. The matrix element squared $|M|^2$ have the factor 4/9 for bq and cq collisions, however there is the some exceeding of the number of quarks over gluons one at middle temperature. We show in the Table II the results of energy losses calculation $\frac{\Delta E}{E}$ for bq and cq collisions by analogy with the Table I also for $p_{b,c} = 8$ GeV.

The calculation gives for summary energy losses: for b quark $(\frac{\Delta E}{E})_{tot}^b = 0.097$ and for c quark $(\frac{\Delta E}{E})_{tot}^c = 0.309$.

In this work we calculated the summary energy losses $(\frac{\Delta E}{E})_{tot}$ of b and c quarks for various energies. However in Sec IV we use the fragmentation function for b and c quark into heavy B and D mesons with accounting of energy losses b and c quarks. Therefore one should evaluate the energy losses in relative values $\frac{\Delta p}{p}$ for various p with accounting of heavy quark masses M . It is convenient for this to use the values $\frac{\Delta E}{p}$, where $\Delta E = \sqrt{p^2 + M^2} - \sqrt{p'^2 + M^2}$. We have with accounting of the next term $\frac{M^4}{p^4}$:

$$\frac{\Delta E}{p} = \frac{\Delta p}{p} + \frac{M^2}{2p^2} \left(1 - \frac{1}{1 - \frac{\Delta p}{p}} \right) - \frac{M^4}{8p^4} \left[1 - \frac{1}{\left(1 - \frac{\Delta p}{p} \right)^3} \right]. \quad (15)$$

Using the calculation of summary values $(\frac{\Delta E}{p})_{tot}$ for various values p and formula (20), we show in Fig. 2 the results of numeral calculations of $\frac{\Delta p}{p}(p)$ for bg , cg and bq , cq collisions ($M_b = 4.75$ GeV, $M_c = 1.3$ GeV). These results we use in Sec. IV for calculation of nuclear modification factor R_{AA} .

IV. CALCULATION OF NUCLEAR MODIFICATION FACTOR R_{AA} FOR b AND c QUARKS

The heavy quarks b and c fragmentize into heavy B and D mesons (look at scheme (5)). In the PHENIX [1] and STAR [2] experiments are observed only single non-photonic electrons in the semileptonic decays of heavy mesons. These fragmentation functions were considered in the number works, for example [19], [20], [21] and others. The fragmentation function contain usually the different parameters for b and c quarks. If the energy of heavy quark decrease before fragmentation, its transverse momentum is shifted by energy loss on the value $\Delta p(p)$. This effect consist in replace of fragmentation function $D(z)$ by effective one $\frac{z'}{z}D(z')$ [22], where:

$$z' = \frac{z}{1 - \frac{\Delta p}{p}}, \quad (16)$$

where $z = \frac{p_H}{p_Q}$. We use here the Peterson fragmentation function [19], which has been used most extensively in applications:

$$D_Q^H(z) = \frac{N_H z(1-z)^2}{[(1-z)^2 + \epsilon_Q z]^2}. \quad (17)$$

The parameter N_H is normalization, and ϵ_Q is $\sim \frac{m_q^2}{m_Q^2}$, the ratio of the effective light and heavy quark masses. Usually is used the traditional values $\epsilon_b \simeq 0.006$ and $\epsilon_c \simeq 0.06$ [23]. However the value $\epsilon_b = 0.006$ is appropriate only when a leading-log (LL) calculation of spectra is used. When NLL (the next to leading level) calculation are used, smaller values of ϵ are needed to fit the data (for bottom quarks cross section). In the work [24] is used the more appropriate value $\epsilon_b = 0.002$ (with FONLL calculation) to fit data. In the present work we use the fragmentation function (17) with this parameter $\epsilon_b = 0.002$. It should be noted also that parameter $\epsilon_c = 0.06$ is extracted from LO fits to charm production data. However this parameter extracted from NLO fits is $\epsilon_c \simeq 0.02$ [25]. This parameter $\epsilon_c = 0.02$ we use also in the fragmentation function (17). The spectra of heavy mesons H after of fragmentation of Q quark is determined by formula:

$$\frac{d\sigma_1}{dp_\perp^H} = \int_{z_{\min}}^1 dz \frac{D_Q^H(z)}{z} \frac{d\sigma}{dp_\perp^Q}, \quad (18)$$

where $p^Q = p^H/z$. The heavy quarks distributions $\frac{d\sigma}{dp_\perp^Q dy}$ for b and c quarks are shown in Fig. 1. If the heavy quark Q lose $\Delta p(p)$ before fragmentation, the spectra of heavy mesons is determined by formula:

$$\frac{d\sigma_2}{dp_\perp^H} = \int_{z_{\min}}^{z_{\max}} \frac{dz}{z} \frac{D_Q^H(z')}{1 - \frac{\Delta p}{p}(z)} \frac{d\sigma}{dp_\perp^Q}. \quad (19)$$

The semi-leptonic decays of heavy mesons dominate the electrons spectrum at RHIC experiment up to $p_\perp \simeq 20$ GeV, and we use the value $z_{\min} \simeq \frac{p_\perp^H}{20}$ for every meanings of p_\perp^H . The value z_{\max} is determined from condition $z' = \frac{p_H}{p - \Delta p} = \frac{z}{1 - \frac{\Delta p}{p}} = 1$. Using this condition and Fig. 2 for $\frac{\Delta p}{p}$, one can build the functions $p_b(p_B)$ and $p_c(p_D)$ and to find z_{\max} for various meanings of p_B and p_D . As we are interested in the total factor $R_{AA}^{b+c}(p_H)$, one should determine the values z_{\max} for the same momentum p_B and p_D of heavy mesons. We write down for example the several values of z_{\max} :

a) for bg and cq collisions:

$$\begin{aligned} P_B = 6 \text{ GeV } z_{\max} &= 0.857 \\ p_D = 6 \text{ GeV } z_{\max} &= 0.658 \\ p_B = 6.9 \text{ GeV } z_{\max} &= 0.86 \\ p_D = 6.9 \text{ GeV } z_{\max} &= 0.678 \\ p_B = 8.5 \text{ GeV } z_{\max} &= 0.85 \\ p_D = 8.5 \text{ GeV } z_{\max} &= 0.71 \end{aligned}$$

b) for bq and cq collisions:

$$\begin{aligned} p_B = 6 \text{ GeV } z_{\max} &= 0.88 \\ p_D = 6 \text{ GeV } z_{\max} &= 0.69 \\ p_B = 6.9 \text{ GeV } z_{\max} &= 0.864 \\ p_D = 6.9 \text{ GeV } z_{\max} &= 0.701 \\ p_B = 8.5 \text{ GeV } z_{\max} &= 0.867 \\ p_D = 8.5 \text{ GeV } z_{\max} &= 0.74 \end{aligned}$$

The spectra of heavy mesons H after of fragmentation without and with energy losses of heavy quarks is determined by formulas (18) and (19) with improving parameters $\epsilon_b = 0.002$ and $\epsilon_c = 0.02$ of Petersons function. The normalization factors N_B and N_D is fixed by the probability that b quark fragment into B meson [19]:

$\int_{z_{\min}}^1 dz D_b^B(z) = 1$ and by analogy for D_c^D . Also the normalization factors N'_B and N'_D for modified fragmentation functions is fixed by condition [22]:

$$\int_{z_{\min}}^{z_{\max}^b} dz \frac{D_b^B(z')}{1 - \frac{\Delta p}{p}(z)} = 1 \quad (20)$$

and similarly for $D_c^D(z')$. However it should be noted that for example the all b quarks with energy losses at bg and bq collisions come from the same spectra $\frac{d\sigma}{dp_\perp}$, therefore one should fix the normalization by condition:

$$N'_B \left[\int_{z_{\min}}^{z_{\max}^{bg}} dz \frac{D_b^B(z')}{1 - \left(\frac{\Delta p}{p}\right)_{bg}(z)} + \int_{z_{\min}}^{z_{\max}^{bq}} dz \frac{D_b^B(z')}{1 - \left(\frac{\Delta p}{p}\right)_{bq}(z)} \right] = 1. \quad (21)$$

We write down here the function $D_b^B(z')$ without normalization factor N'_B , and from here we determine normalization factor N'_B . From analogous condition for c quarks (with $D_c^D(z')$ and corresponding values $z_{\max}^{g,q}$ we determine the factor N'_D). These conditions correspond to value of the nuclear modification factor without energy losses - $R_{AA}^{b+c} = 1$. Thus the total factor $R_{AA}^{b+c}(p_\perp^H)$

for b and c quarks with energy losses it is convenient to write in the form:

$$R_{AA}^{b+c}(p_{\perp}^H) = \frac{N'_B(A_{bg} + A_{bq}) + N'_D(A_{cg} + A_{cq})}{N_B A_b + N_D A_c}. \quad (22)$$

We use here the designation in numerator:

$$A_{bg} = \int_{z_{\min}}^{z_{\max}^{bg}} \frac{dz}{z} \frac{D_b^B(z')}{1 - \left(\frac{\Delta p}{p}\right)_{bg}(z)} \frac{d\sigma}{dp_{\perp}^b},$$

and A_{bq} correspond to $A_{bg \rightarrow bq}$.

We use also:

$$A_{cg} = \int_{z_{\min}}^{z_{\max}^{cg}} \frac{dz}{z} \frac{D_c^D(z')}{1 - \left(\frac{\Delta p}{p}\right)_{cg}} \frac{d\sigma}{dp_{\perp}^c},$$

and A_{cq} correspond to $A_{cg \rightarrow cq}$.

The designation in denominator is

$$A_b = \int_{z_{\min}}^1 \frac{dz}{z} D_b(z) \frac{d\sigma}{dp_{\perp}^b},$$

and

$$A_c = \int_{z_{\min}}^1 \frac{dz}{z} D_c(z) \frac{d\sigma}{dp_{\perp}^c}.$$

In these formulas the fragmentation functions D_b and D_c we write down also without normalization factors.

However in experiments are observed only non-photonic electrons in the semileptonic decays of heavy mesons. It was noted [3], that apparently the electrons from b mesons dominate the spectrum for $p_{\perp} \geq 4$ GeV. But recently the contribution of B meson decays to non-photonic electrons in $p+p$ collisions at $\sqrt{s}=200A$ GeV has been measured using azimuthal correlation between non-photonic electrons and hadrons [26]. The extracted B decay contribution is approximately 0.5 at $p_{\perp} \geq 5$ GeV/c. The $\frac{e_B}{e_B+e_D}$ ratio is found to be 0.52 ± 0.03 . We use this result for calculation of factor R_{AA}^{b+c} by formula (22), assuming the absence of the exceeding of electrons spectrum from B mesons. We use the formulas (21), and (22) for calculation of factors N'_B and N'_D and also of nuclear modification factor $R_{AA}^{b+c}(p_{\perp}^H)$. We use the Figures 1 and 2 for numeral calculation on the z integrals for every of fixed meanings $p_{\perp}^B = p_{\perp}^D$. We calculate also the corresponding factors N_B and N_D without energy losses. We show here the results of numeral calculation of factor $R_{AA}^{b+c}(p_{\perp})^H$ for example at $p_{\perp}^B = p_{\perp}^D = 6.9$ GeV/c in accordance with designations in formula (22) for every member in this formula: $N'_B = \frac{1}{14.55+14.147}$, $A_{bg} + A_{bq} = 0.172+0.175$, $N'_D = \frac{1}{2.646+2.742}$, $A_{cg} + A_{cq} = 0.02+0.025$, and $N_B = \frac{1}{14.312}$, $A_b = 0.277$, $N_D = \frac{1}{3.377}$, $A_c = 0.18$.

Table III: The nuclear modification factor for collision b and c quark with gluons only and with gluons and light quarks

p_{\perp}^H (GeV/c)	R_{AA}^{Qg}	$R_{AA}^{Q(g+q)}$
5.5	0.22	0.235
6	0.235	0.251
6.9	0.265	0.281
8	0.295	0.315
8.5	0.315	0.335

In result we have the nuclear modification factor for $p_{\perp}^H = 6.9$ GeV/c from formula (22): $R_{AA}^{b+c}(p_{\perp}^H = 6.9$ GeV/c) = 0.281

We give in the Table III the results calculations of factor R_{AA}^{b+c} for some meaning p_{\perp}^H at collisions of b and c quarks with gluons only (R_{AA}^{Qg}) and with gluons and light quarks ($R_{AA}^{Q(g+q)}$)

We see that accounting of b and c quark collisions also with light quarks gives exceeding of value R_{AA}^{b+c} in less than 10 per cent. We show in Fig. 3 the comparison of nuclear modification factor $R_{AA}^{b+c}(p_{\perp}^H)$ at 5.5 GeV/c $\leq p_{\perp}^H \leq 8.5$ GeV/c in the present model with experimental data for central $Au + Au$ collisions at $\sqrt{s} = 200A$ GeV [2]. We have agreement in afore-cited region of p_{\perp}^H . The not great increase of R_{AA}^{b+c} with increase of p_{\perp}^H apparently is also in experiment. We do not calculate this factor R_{AA}^{b+c} for too low meanings of p_{\perp}^H , where the Petersons fragmentation functions in form (17) apparently inapplicable [19].

V. CONCLUSION

We investigate in this work the collision energy losses of b and c quarks at hard collisions with gluons and light quarks in QGP produced in central $Au + Au$ collisions at RHIC energy. We assume that heavy quarks not come to an equilibrium with partons in plasma. The hard collisions are described by formulas of perturbative QCD (pQCD) at large enough momenta transferred. The hard collisions in QCD limited by momenta $Q^2 \simeq \Lambda^2$, where in the pQCD we have Landau pole in the coupling constant $\alpha_s(Q^2)$ at $Q^2 = \Lambda^2$ (the $\Lambda \simeq 200$ MeV is dimensional constant of QCD). In this work we use the analytic models for the running coupling in QCD, which satisfied to conditions of analyticity in local Quantum Field Theory (look at Sec. II). We use here the analytic model [8], [9] with invariant charge in the form (4), which have no Landau pole and describe also the asymptotic at large momenta transferred $Q^2 > \Lambda^2$. In Sec. III we calculate the energy losses of b and c quarks at hard collisions with light partons at momenta transferred $Q^2 \geq \Lambda^2$. We show in Fig. 2 the results of calculations of summary losses $\Delta p/p$ of b and c quarks with momentum $p_{b,c}$ at hard collisions with gluons and also with light quarks. These results we use in Sec. IV for calculation of the total nu-

clear modification factor R_{AA}^{b+c} . We use the fragmentation function for b and c quarks into B and D mesons with accounting of summary losses $\Delta p/p$ and also fragmentation functions without energy losses. We use here the Petersons fragmentation function [19], which has been widely adopted in analysis determining the "hardness" of heavy quarks fragmentation functions. Its attraction being that it has only one free parameter, ϵ_Q , for each heavy quark Q . But we use the Petersons fragmentation functions with improving parameters ϵ_Q for b and c quarks to fit the data, as has been mentioned in Sec. IV. We take into account to the equal degree the energy losses of b and c quark at calculations. One should not neglect by b quark contribution, thought it is less than c quark one. We take into account also the relative contribution of B decays $\simeq 50$ percents in non-photonics electrons [26]. We show in Fig 3 the comparison of factor R_{AA}^{b+c} for heavy mesons B and D in the present model with experimental

data [2]. These data show the analogous suppression of nonphotonics electrons in the region of middle meanings of p_\perp . It is interesting to compare the energy losses of heavy quarks and nuclear modification factor with others models. We plan also the analogous calculation with others fragmentation functions.

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Figure 1: The differential cross section of charm (the upper curve) and bottom (the lower curve) quarks in pp collisions at $\sqrt{s_{NN}} = 200$ GeV from [3], [5]

Figure 2: The relative the energy losses $\Delta p/p$ b and c quarks at various momenta. The upper curves correspond to c quark for cg collisions (the curve 1) and for $cg + cq$ (the curve 2). The lower curves correspond to analogous collisions for b quarks (the curve 3 and 4)

Figure 3: The comparison of nuclear modification factor R_{AA}^{b+c} in the present model at middle meaning of p_{\perp} (the solid curve) with experimental data of STAR collaboration [2] for nonphotonic electrons.





